MATHEMATICS

This section of the test covers mathematics. Even though Lesson 1 starts with simple arithmetic, do not skip any portions of the testing. There are also numerous questions dealing with geometry and algebra.

LESSON 1

ARITHMETIC

Sum. Difference. Product. Quotient. Do you know the meanings of these words? If you don't you may be unable to solve some mathematics problems. Learn the meanings of mathematical terms as well as how to use them in arithmetic.

Addition is the totaling of two or more numbers to find their sum. The numbers which are added are called the addends. The sign of addition is a plus sign (+).

```
EXAMPLE:     2132}  addends
            +1562}  addends
             3694   sum
```

The equal sign (=) may be used to indicate amounts of equal value.

EXAMPLE: 3 + 2 = 5

Subtraction is the opposite of addition. It is the process of "taking away" to find the difference. The minus sign (-) indicates subtraction .

```
EXAMPLE:     3694   minuend
            -1562   subtrahend
             2132   difference
```

Multiplication is a short way of adding. The multiplication or "times" sign (x) is used. The answer is called the product.

```
EXAMPLE:     48     multiplicand
            x15     multiplier
            240
            48
            720     product
```

Division is the reverse of multiplication. The division sign (÷) means "divided by." For example, 6 ÷ 2 = 3. Another way to indicate division is shown below.

```
EXAMPLE:                147    quotient
          divisor     5)735    dividend
                        5
                        23
                        20
                         35
                         35
```

EXERCISE 1

A. *Addition* – Find the sum of the addends.

1. 7 +9	2. 5 +4	3. 78 +33	4. 58 +95	5. 331 + 27
6. 3,986 +4,271	7. 5,930 +7,826	8. 97,413 +56,208		9. $1.98 + .45

B. *Subtraction* – Find the difference

1. 9 -5	2. 65 -36	3. 894 -576	4. 321 -179	5. 4,658 -3,219
6. 7,163 -4,258	7. 39,674 -17,826	8. $10.98 - 5.67		9. $795.86 - 436.17

C. *Multiplication* – Find the product.

1. 13 x 3	2. 47 x 8	3. 695 x 4	4. 6,957 x 314	5. $84.53 x 72

D. *Division* – Find the quotient.

1. 3) 24
2. 7) 427
3. 28) 26,936
4. 769) 96,125
5. 7) $37.24

LESSON 2

FRACTIONS

The study of fractions involves working with whole numbers which have been divided into two or more parts. The parts are known as fractional parts of the whole number. *Remember:* the numerator (top number) tells you *how many*; the denominator (bottom number) tell you the *number of parts* into which the whole number has been divided.

EXAMPLE: The fraction 5/6 means the whole number has been divided into 6 parts, and that this fraction is dealing with 5 of those parts.

To add fractions, with like denominators, just add the numerators and place the sum over the same denominator.

EXAMPLE: $\frac{2}{3} + \frac{2}{3} + \frac{1}{3} = \frac{5}{3}$

When the numerator is larger than the denominator, it indicates that more than one whole number is represented. This is called an *improper fraction*. An improper fraction should be reduced to its simplest form. To find how many whole numbers are in an improper fraction, divide the denominator into the numerator.

EXAMPLE: $\frac{3}{3}$ is equal to 1. Likewise, $\frac{5}{3}$ is equal to 1 2/3.

Fractions may be changed by multiplying or dividing both the numerator and the denominator by the same number. This procedure allows you to change fractions with unlike denominators into fractions with like denominators. The fractions can then be added or subtracted. The value of the fraction is not changed.

EXAMPLE: To add ½ and ⅜, it is necessary to change ½ into eighths. Multiply the numerator and the denominator by the number necessary to change ½ into eighth – in this case, 4. 1 x 4 = 4; 2 x 4 = 8. ½ = 4/8.

$$\frac{4}{8} + \frac{3}{8} = \frac{7}{8}$$

EXAMPLE: To change 2/6 into thirds, divide both the numerator and the denominator by 2. 2 ÷ 2 = 1; 6 ÷ 2 = 3. 2/6 = 1/3.

This is also referred to as *reducing* a fraction. When doing problems involving fractions, the final answer should be always be reduced to the simplest form.

EXERCISE 2

A. Add the following fractions.

1. $\frac{2}{4} + \frac{1}{4}$
2. $\frac{7}{9} + \frac{2}{9}$
3. $\frac{2}{6} + \frac{3}{6}$
4. $\frac{3}{10} + \frac{5}{10} + \frac{7}{10}$
5. $\frac{7}{4} + \frac{3}{4} + \frac{2}{4}$
6. $\frac{5}{16} + \frac{8}{16} + \frac{7}{16}$
7. $\frac{6}{20} + \frac{9}{20} + \frac{5}{20}$

B. Reduce the following fractions to their simplest form.

1. $\frac{25}{5}$
2. $\frac{15}{10}$
3. $\frac{12}{4}$
4. $\frac{20}{16}$
5. $\frac{20}{20}$
6. $\frac{18}{12}$
7. $\frac{16}{5}$
8. $\frac{22}{9}$
9. $\frac{35}{9}$
10. $\frac{7}{3}$

C. Find the like denominators, add the fractions, and reduce the answers to the simplest form.

1. $\frac{1}{6} + \frac{1}{3}$
2. $\frac{1}{5} + \frac{5}{8}$
3. $\frac{2}{3} + \frac{5}{9}$
4. $\frac{3}{8} + \frac{1}{4}$
5. $\frac{7}{9} + \frac{10}{18}$
6. $\frac{18}{24} + \frac{1}{4}$
7. $\frac{3}{7} + \frac{3}{14}$
8. $\frac{4}{5} + \frac{9}{10}$
9. $\frac{2}{9} + \frac{3}{18}$
10. $\frac{7}{8} + \frac{3}{4}$
11. $\frac{2}{3} + \frac{5}{6}$
12. $\frac{7}{11} + \frac{7}{22}$

LESSON 3

Common Fractions – Subtraction and Multiplications

The subtraction of fractions with like denominations is just the opposite of addition. When fractions have like denominators, the numerator of the subtrahend is subtracted from the numerator of the minuend, and the results is placed over the like denominator and reduced to its simplest form.

EXAMPLE: $\frac{5}{9} - \frac{3}{9} = \frac{2}{9} = \frac{1}{3}$

Convert unlike denominators to like denominators and proceed as discussed above.

EXAMPLE: $\frac{1}{8} = \frac{1}{16}$ Multiply the numerator and denominator of $\frac{1}{8}$ by 2 to convert the fraction to sixteenths. Then subtract: $\frac{2}{16} - \frac{1}{16} = \frac{1}{16}$

The multiplication of fractions is not concerned with common denominations. To multiply a whole number by a fraction for a fraction by a whole number by a whole number to a fraction by placing it over 1, and then multiply the numerators times the numerators and the denominators times the denominators.

EXAMPLE: $10 \times \frac{1}{3} = \frac{10}{1} \times \frac{1}{3} = \frac{10}{3}$

To multiply a fraction by a fraction, multiply the numerators to find the new numerator and the denominators to find the new denominator.

EXAMPLE: $\frac{3}{4} \times \frac{2}{3} = \frac{5}{12}$

To multiply mixed numbers (1 2/3, etc.), convert the mixed numbers to improper fractions and work as for fractions. As you can see, all multiplication involving fractions can be done as though multiplying fractions by fractions.

EXAMPLE: 2 ½ x 3 ¼ Convert to: $\frac{5}{2} \times \frac{13}{4} = \frac{65}{8}$ reduce: 8 1/8

EXERCISE 3

Work the following word problems using Lesson 3 as a guide. Reduce each answer to its simplest form.

1. George and Saul bought a car together. George owned 5/8 of the car; Saul owned 3/8. How much more does George own than Saul?

2. Roberta bought 7/8 yard of dress material. Lois needed only ¾ yard. How much more did Roberta buy than Lois?

3. Raul bought 5/9 acre of land to add to the 2/3 acre he already owned. How much acreage did Raul then own?

4. If Carla had 1 2/16 pound of butter and Julia had 3/8 that much, how much did Julia have?

5. Anna bought 13 of the ¼ acre lots. How many acres did she buy?

6. Louis farmed 2/3 of a 7/8 acre tobacco allotment. How much land did he farm?

LESSON 4

Common Fractions – Cancellation and Division

A shortcut method called *cancellation* makes it easier to multiply and divide fractions. As you can see by studying the example below, cancellation involves dividing numerators or denominators by a factor common to both. This process cancels out the common factors in both.

EXAMPLE: Long method $\frac{2}{5} \times \frac{5}{8} = \frac{10}{40}$ or $\frac{1}{4}$

Cancellation method $\frac{2}{5} \times \frac{5}{8} = \frac{1}{4}$

In dividing whole numbers by fractions, or fractions by whole numbers, *every whole number is considered to be a fraction.* For example, 5 is considered to be 5/1. To divide by a fraction, invert the divisor and multiply.

EXAMPLE: $12 \div \frac{2}{3} = \frac{12}{1} \times \frac{3}{2}$ $\qquad \frac{\overset{6}{12}}{1} \times \underset{1}{\frac{2}{3}} = \frac{16}{1}$ or 18

EXAMPLE: $\frac{2}{3} \div 12 = \frac{2}{3} \times \frac{1}{12}$ $\qquad \frac{\overset{1}{2}}{3} \times \frac{1}{\underset{6}{12}} = \frac{1}{16}$

The same rules apply to all fractions and mixed numbers.

EXAMPLE: $\frac{3}{8} \div \frac{3}{4} = \frac{3}{8} \times \frac{4}{3}$ $\qquad \frac{\overset{1}{3}}{\underset{2}{8}} \times \frac{\overset{1}{4}}{\underset{1}{3}} = \frac{1}{2}$

EXAMPLE: $4\frac{2}{3} \div 7 = \frac{14}{3} \times \frac{1}{7}$ $\qquad \frac{\overset{2}{14}}{3} \times \frac{1}{\underset{1}{7}} = \frac{2}{3}$

One of the major advantages of cancellation, in addition to being a time-saver, is that by working with smaller numbers, often one digit instead of two, the opportunity for careless error is significantly reduced. Whenever possible, it is to your advantage to streamline operations before beginning the actual computations.

EXERCISE 4

Work these problems using Lesson 4 as a guide.

1. Lucille walks 9/16 mile to work. Georgia walks only 1/3 as far. How far does Georgia walk?

2. Eight acres of land were divided into 1/3 acre plots. How many plots were there?

3. If 4/5 acre of land is to be snared equally by two brothers, how much will each have?

4. A cake weighing 2 ¾ pounds is to be shared by 11 children at a birthday party. How much will each child's share be?

5. A 12-pound bag of flour will make how many 2-ounce biscuits?

6. If 1 yard of material will make 6 ¾ washcloths, how many yards will be needed for 24 washcloths?

7. A bus travels 64 city blocks in an hour. How far will a bus go traveling at 7/8 the speed?

LESSON 5

Decimal Fractions

Decimal fractions represent the fractional part of a whole number. The placement of the decimal point indicates the denominator.

This decimal system is expressed as follows:

0.1 = tenths
0.01 = hundredths
0.001 = thousandths
0.0001 = ten-thousands
0.00001 = hundred-thousandths
0.000001 = millionths

To add and subtract decimal fractions, the decimal numbers must be placed in columns so that all decimal points align.

EXAMPLE:

```
  4.621        4.621
+ .025        -2.954
  4.646        1.677
```

The regular rules of multiplication apply to all problems dealing with decimal fractions. The number of decimal places in the answer is the sum of the number of decimal places in the multiplier and the multiplicand.

EXAMPLE:

```
   2.4
 x .57
  120
 168
1.800
```

The division of decimals is done in the same way as the division of whole numbers. However, the decimal point must be placed correctly in the quotient. This is done by moving the decimal point in the divisor to the right to make it a whole number. The decimal point in the dividend is moved to the right the same number of places. The decimal point in the quotient is placed over the new location of the decimal point in the dividend. Zeros may need to be annexed to the dividend.

EXAMPLE:

```
       ___.__
1.2.) 3.6.5
```

EXERCISE 5

Work the problems, using Lesson 5 as a guide. Circle the correct answer.

1. In one day, three bricklayers worked 8.5 hours, 7.25 hours, and 9.75 hours respectively. What was the average number of hours worked?

A. 8 hours	C. 8.3 hours
B. 8.5 hours	D. 25.50 hours

2. Mrs. Mendez drives 16.4 miles each way to work and back. She works a six-day week. How many miles does she drive each work-week?

A. 19.68 miles	C. 196.8 miles
B. 5.46 miles	D. 164.0 miles

3. José's truck gets 14.5 miles to a gallon of gasoline. A round trip to market is 78.3 miles. If gasoline costs 35 cents per gallon, how much does each trip cost?

A. $1.89	C. $2.24
B. $2.89	D. $18.90

4. Janet Anderson drove her car ¾ as fast as Sue Granger drove hers. If Sue Granger drove 35 miles in one hour, how far did Janet Anderson drive?

A. 11.66 miles	C. 75 miles
B. 233 miles	D. 26.25 miles

5. If ground beef is $1.15 per pound and Angela has $3.45, how may pounds can she buy?

A. 3 pounds	C. 3.93 pounds
B. .3 pounds	D. 39.3 pounds

6. Sam, Harry, and Lew picked 48.9 quarts of cherries in 8 hours. How many quarts did each take home if the total amount was divided equally?

A. 24.5 quarts	C. 16.3 quarts
B. 146.7 quarts	D. 6.1 quarts

7. Three people earned a total of $46.20 at a cookware party. If they divide the profits equally, how much will each person get?

A. $17.40	C. $154.00
B. $15.40	D. $14.40

LESSON 6

Percent

Percent means "hundredths." Thus, 8 percent means 8/100, or 8 of 100 parts. We use the symbol % to mean percent; 8%, 8/100, and .08 all represent the same value.

To change a percent to a decimal fraction, simply remove the percent symbol and move the understood decimal point two places to the *left.*

EXAMPLE: 25% = .25; 7% = .07

62.5% = .625; 2.5% = .025

To change a decimal fraction to a percent, simply move the decimal point two places to the *right* and add the percent symbol. For example, .625 = 62.5%, .47 = 47%, and .2 = 20%.

To change a common fraction to a percent, divide the numerator by the denominator and continue the division to at least two decimal places. Zeros often will need to be annexed to the dividend to complete the division. For example, to change 1/8 to a percent:

```
   _.125_ = 12.5% or 12 ½ %
8)  1.000
    __8
      20
      16
       40
       40
        0
```

The use of percent is helpful in solving many kinds of problems. It is a tool that frequently can be used in everyday transactions. Many retailers advertise discounts or sales in terms of percent. It is useful to know how to determine a dollar figure. To find a percent of a number, multiply the number by the decimal fraction equivalent to the percent. To find what percent one number is of another, divide the whole into the part and change the decimal into a percent. To find a number when a percent of it is given, change the percent to a decimal or a fraction and divide this into the given number. *Percent of increase* is computed by finding the amount of increase and dividing it by the original amount. *Percent of decrease* is computed by finding the amount of decrease and dividing it by the original amount.

EXERCISE 6

Work the problems, using Lesson 6 as a guide. Circle the correct answer.

1. A general rule in figuring a family budget is that rent should not exceed 25% of one's monthly income. If Sam earns #12,360 per year, what limit should he set on his monthly rent?

A. $176.50	C. $1,030.00
B. $1,350.00	D. $257.50

2. Maria got a 6% raise. Her pay last month (before the raise) was $880. With the 6% raise, what will be her pay next month?

A. $1,680	C. $620
B. $932.80	D. $725

3. Charles bought a table for $65 and sold it for $84.50. What was his percentage of profit?

A. 70%	C. 30%
B. 15%	D. 25%

4. If a share of stock costing $45.00 lost 3/8% of its value, how much is it now worth?

A. $0.16875	C. $44.83
B. $43.31	D. $16.88

5. Laura is a salesperson in a department store. She makes 8% commission on all the clothes she sells. Her sales last week were $2,650.00. How much did she make?

A. $220.00	C. $330.00
B. $210.00	D. $212.00

6. In one day Henry picked 40 boxes of oranges. Ed picked 80% as many boxes as Henry. How many boxes of oranges did Ed pick?

A. 28	C. 34
B. 32	D. 36

7. A survey showed that a factory only employed one woman for every ten men. What percent of their work force is female?

A. 10%	C. .10%
B. 1 %	D. 1.0%

LESSON 7

Set Theory

Modern mathematics has its own language of terms and symbols. A *set* is any group or collection of objects. Objects which belong to a set are called *members* or *elements* of the set. A set is identified by a capital letter, and braces enclose the elements of a set.

EXAMPLE: The whole numbers 1 through 5 are a set. This could be written A = {1, 2, 3, 4, 5}. Each of the individual numbers is an element of the set.

If the members of a set may be counted, the set is called a *finite set.* The set in the example above is a finite set. A set having an unlimited number of members is called an *infinite set.* All of the counting numbers would make up an infinite set. *Equal sets* contain exactly the same elements.

{1, 2, 3} = {3, 2, 1}

Sets containing the same number of elements are called *equivalent sets.*

{3, 5, 7} ≈ {2, 4, 6}

A set containing no elements is termed an *empty* or *null set* and is represented by the Greek letter ø (phi), or by empty braces. A set whose elements are contained in a larger set is said to be a *subset* of that set.

EXAMPLE: {1,3,5} is a subset or {1,2,3,4,5}. This is written {1,3,5} ⊏ {1, 2, 3, 4, 5}

Overlapping sets have some elements in common. *Disjoint sets* have no elements in common.

The *intersection* of two is the set holding all the common elements of the two sets. If A = {a, b, c, d, e} and B = {a,c,f,g}, the intersection of the sets is {a,c}. This is written A ⊓ B = {a,c}.

The *union* of two sets is the set holding all the different elements from both sets, with each element listed only once.

EXAMPLE: A = {u, v, w, x}

B = {y,z}
A∪B = {u, v, w, x, y, z}

Note that these are disjoint sets.

EXERCISE 7

1. Using braces, name the set of the days of the week.

____

2. The set of the first five letters of the alphabet is

____

3. List the set of odd numbers 1 through 15.

____

4. Is the set of all odd numbers infinite? ________________________________

5. Is the set of the names of the months of the year infinite? ________________

6. What kind of set is Monday and Tuesday in relation to the set of the days of the week?

____

7. Tell whether each of the following are equal sets, equivalent sets, null sets, overlapping sets, or disjoint sets. Remember that a set may be more than one of the above.

   a. A = {all teen-agers over 35} ________________________________
   b. {52, 39, 76} and {39, 76, 52} ______________________________
   c. {60, 3, 17} and {21, 4, 6} ________________________________
   d. {dog, tiger, horse} and {cow, lion, dog, bird} ___________________

8. Write the answer set for each of the following

   a. {6,8,10, 12} $\cup$ {2, 4, 6, 8, 12, 14} = __________________________
   b. {football, baseball, basketball} $\sqcap$ {soccer, volleyball, baseball} =

      ____

   c. {1, 4, 12, 3, 7, 21} $\cup$ {3, 10, 21, 5, 8} = _______________________
   d. {apples, oranges, bananas, pears} $\cup$ {raisins, grapes, pears, apples} =

      ____

LESSON 8

Algebra

The formulas used in the preceding lessons involved some known quantities and some unknown quantities. The branch of mathematics which deals with the relationship between known and unknown quantities is *algebra.* The statement which expresses the relationship of the numbers and symbols in an algebra problem is called an *equation.*

Algebra uses the same operation signs that are used in arithmetic: +, —, ×, ÷. Sometimes multiplication is not indicated by a sign but is understood. Whenever a number appears before a letter or before a term in parentheses, it is understood that multiplication is to take place. For example, 5y means 5 times y; 4(6) means 4 times 6; xy means x times y. Another way of indicating multiplication is by the use of a raised dot $5 \cdot b$ means 5 times b.

The solution of an algebraic equation requires that you find the value of the unknown. The equation will usually have to be restructured so that the unknown is on one side and the known parts are on the other. Remember that whatever operation is performed on one side of an equation must also be performed on the other. For example, to solve the equation $x - 2 = 6$, x can be isolated by adding 2 to each side of the equation. Then $x = 6 + 2$, or $x = 8$. Where the unknown is multiplied by a number, both sides of the equation must be divided by that number, both sides of the equation must be divided by that number to solve the equation. Where the unknown is divided by a number, both sides of the equation must be multiplied by that number to solve the equation. For example, in the equation $8x = 64$, both sides of the equation must be divided by 8. Thus, $x = 8$. If $x/3 = 15$, then $x = 45$.

Some equations may be solved by substituting given values for unknowns in an equation. For example, to find the value of the statement $8x - 3y + 7z$ where $x = 8$, $y = 4$, and $z = 5$:

$$\begin{aligned} 8x - 3y + 7z &= 8(8) - 3(4) + 7(5) \\ &= 64 - 12 + 35 \\ &= 87 \end{aligned}$$

To find the value of the statement $\frac{x}{2} - y + 3z$ using the values above:

$$\begin{aligned} \frac{8}{} - 4 + 3(5) &= 4 - 4 + 15 \\ &= 15 \end{aligned}$$

EXERCISE 8

Substitute the given values for the unknowns and solve the equations. Circle the correct answer.

Given Values: $x = 5$, $y = 6$, $a = 4$, $b = 3$

1. $6x =$
   a. 36 c. 6
   b. 30 d. 24

2. $x + y =$
   a. 30 c. 11
   b. 10 d. 12

3. $3ab =$
   a. 36 c. 10
   b. 15 d. 12

4. $4a(3b) =$
   a. 12 c. 16
   b. 36 d. 144

5. The value of $x^2 - 6 + 2b$ is
   a. 25 c. 45
   b. 11 d. 31

6. The value of $x(y - 2) + ab$ is
   a. 21 c. 38
   b. 35 d. 32

7. $\frac{x}{5y} + \frac{a}{5b} =$
   a. $\frac{13}{30}$ c. $\frac{13}{60}$
   b. $\frac{17}{30}$ d. $\frac{1}{3}$

8. $7 \times Y =$
   a. 35 c. 210
   b. 30 d. 140

9. $a^2 - b^2 =$
   a. 7 c. -7
   b. 4 d. -4

10. $x(a - b) + y =$
    a. -6 c. 12
    b. 14 d. 11

11. $\frac{y}{ab} =$
    a. ¼ c. ½
    b. 5/12 d. ¾

12. $y(x - a) + b =$
    a. 9 c. 10
    b. 3 d. 8

13. $x^2 + y^2 + b^2 - a^2 =$
    a. 54 c. 77
    b. 70 d. 74

14. $4(xy) - 2(ab) =$
    a. 18 c. 108
    b. 144 d. 96

Solve the following equations

15. $3x - 5 = 13$

16. $5(x + 4) = 4(x + 6)$

17. $5x - 2 = 3x + 2$

18. $2x + 8 = x + 14$

19. $3x - 2 = 2(x+2)$

20. $7x - 4 = 3x$

21. $4x - 12 = 12$

22. $3(x + 7) = 27$

23. $x + 12 = 2x + 3$

24. $3x = x + 12$

25. $5x + 2(x - 2) = x + 14$

26. $9x - 7 = x + 9$

LESSON 9

Formulas

A formula is a rule or principle which applies to be a given situation.

Example: *I = prt,* or *Interest = principal x rate of interest x time.*

To use a formula, substitute the values given in a problem for the letter symbols in the formula.

Example: If the rate of interest charged for borrowing money is 8% per year, how much is the annual interest on $150?

Formula *I = prt*
I = $150 x 8% x 1 year
I = $12.00

The perimeter of a figure is computed by adding the lengths of the sides. The perimeter of a triangle is equal to the sum of the lengths of its three sides.

$P = S_1 + S_2 + S_3$

The perimeter of a rectangle is equal to the sum of its four sides.

P = 2L + 2W (2 lengths plus 2 widths)

The circumference of a circle is approximately 3 1/7 times the length of the diameter of the circle. The formula $C = \pi D$ uses the Greek term *pi* which is represented by the symbol π. This represents the constant multiplier of 3 1/7, which can be converted to the improper fraction 22/7 or the decimal fraction 3.1416. (In the problems in Exercise 7 and following exercises, the decimal fraction 3.1416 has been used, because it yields greater accuracy than the improper fraction 22/7. However, you should practice using both.)

The formula for the circumference of a circle can also be written $C = 2 \pi r$, where r represents the radius of the circle (half the diameter).

Example: A circle has a radius of 6 inches. The circumference is:
C = 2 x 3.1416 x 6
C = 6.2832 x 6
C = 37.6992

EXERCISE 9

Work the problems, using Lesson 9 as a guide. Circle the best possible answer.

1. Arthur and Sue bought a refrigerator for $350.00. The interest was 12% for one year. How much did they pay for the refrigerator including the interest?

a. $42.00
c. $392.00
b. $350.00
d. $360.00

2. A field which needs a new fence measures .5 mile by .4 mile by .3 mile. At a cost of $1.80 per foot, how much will the fence cost?

a. $831.60
c. $1,188.00
b. $11,404.80
d. $8,316.00

3. Jim planted border grass around his backyard. The yard measured 45 feet by 25 feet. If three clumps of grass were planted to the running foot, how many clumps of grass did Jim plant?

a. 420
c. 4,500
b. 140
d. 13,500

4. What is the difference between the circumferences of two circles if the diameter of one measures 4 inches and the diameter of the other measures 5 inches?

a. 3.1416 inches
c. 1 inch
b. 31.416 inches
d. 9 inches

5. Find the circumference of a circle whose radius is 15 feet.

a. 30 feet
c. 942.480 feet
b. 94.248 feet
d. 47.124 feet

6. If the circumference of a circle is 65.9 inches, what is the diameter?

a. 207 inches
c. 10.5 inches
b. 210 inches
d. 21 inches

7. The Riveras borrowed $1,500.00 at the rate 14% per year. At the end of two years they repaid the loan. How much was the interest?

a. $42.00
c. $420.00
b. $210.00
d. $240.00

LESSON 10

Formulas and Equations

To find the area of any rectangle, multiply its length by its width. $A = lw$. (Be sure the length and width are expressed in the same units.)

Example: A rectangle 3 feet long and 2 feet wide has an area of 6 square feet.
$A = 3 \times 2 = 6$ sq. ft.

To find the area of a triangle, multiply half the base by the height. $A = ½\ bh$. In the example below, the base is 10 feet and the height is 6 feet.

$A = (½)(10)(6)$
$A = 5 \times 6$ or 30 sq. ft.

To find the area of any circle, square the radius and multiply by pr (π). $A = \pi r^2$. The radius is ½ of the diameter. In a circle with a diameter of 14, the radius is 7. $A = \pi r^2$ or $(22/7) \times 7 \times 7$. Its area is 1154 square units. Area is always expressed in square units.

To find the volume of a rectangular solid (container), multiply the length times the width times the height.. $V = lwh$. Imagine a rectangular box with the length = 6, the width = 4, and the height = 3. $V = 6 \times 4 \times 3$ or 72 cubic units. Volume is always expressed in cubic units.

To find the volume of a cylinder, use this formula: $V = \pi r^2 h$. Here, π is multiplied by the square of the radius of the circular base. That product is the multiplied by the height of the cylinder. A cylinder (a coffee can) that is 7 inches tall, with a diameter of 5 inches (2.5 inch radius), has a volume of 137.5 cubic inches.

$V = 22/7) (2.5) (2.5) (7) = (22) (6.25)$
$V = 137.5$ cu, in.

Formulas can be presented in many different forms, depending upon what information is known and what you wish to find. Here are some variations of common formulas:

$A = lw$ or $l = A/w$ or $w = A/l$

$V = lwh$ or $l = V/wh$ or $w = V/lh$ or $h = V/lw$

$A = ½\ bh$ or $h = 2A/b$ or $b = 2A/h$

$C = \pi D$ or $D = C/\pi$

$I = prt$ or $p = l/rt$ or $r = 1/pt$ or $t = l/pr$

EXERCISE 10

Work the problems, using Lesson 10 as a guide. Use decimal equivalents in formulas whenever possible. Circle the best possible answer.

1. If 1.8 pounds of grass seed will cover 5 square yards, how many pounds will Ricardo need to seed his front lawn, which measures 90 feet x 60 feet?

a. 45 lbs	c. 216 lbs.
b. 120 lbs	d 5,400 lbs

2. What is the area of a triangle which has a base of 9.4 inches and a height of 5.5 inches?

a. 25.85 sq in.	c. 258.5 sq in
b. 51.70 sq in	d. 5.17 sq in

3. Find the area of a circle whose diameter is 8 2/5 inches.

a. 211.68 sq in	c. 70.56 sq in
b. 17.64 sq in	d. 55.42 sq in

4. Richard had a new concrete driveway pouted. It measured 15 feet by 45 feet by 4 inches. If poured concrete cost $20.50 per cubic yard, how much did the driveway cost?

a. $170.83	c. $225.00
b. $8.40	d. $27.00

5. If a silo is 100 feet high and 20 feet in diameter, what is its capacity in cubic yards?

a. 31,416	c. 10,313
b. 1,163.6	d. 116.35

6. If a school's play yard has 37,500 sq ft. of play area, and the width is 50 yards, how long is the field?

a. 50 yards	c. 150 feet
b. 150 yards	d. 250 feet

7. A gallon of paint will cover 200 square feet. How much paint is required to do a wall which is 30 feet high and 40 feet wide?

a. 60 gallons	c. 6 gallons
b. 20 gallons	d. 14 gallons

8. What is the capacity of a refrigerator that measures 60 inches high and is 2 ½ feet wide?

a. 12.5 cu ft	c. 1,800 cu in
b. 900 cu in	d. cannot be determined

LESSON 11

Polygons

A *polygon* is a closed figure formed by straight line segments all in the same plane. The line segments intersect only at their end points, and only two line segments intersect at any one point.

A three-sided polygon is a *triangle.* The interior angles of a triangle add up to 180°. The *perimeter* of all triangles is determined by adding the lengths of all three sides. To determine the *area* of all triangles, use this formula: A = ½ bh, or area is equal to half the length of the base times the height.

EQUILATERAL TRIANGLE – all angles equal and all sides equal
ISOSCELES TRIANGLE – two sides equal and two angles equal
SCALENE TRIANGLE - no sides equal and no angles equal
RIGHT TRIANGLE – one 90° angle

Polygons are named for the number of sides they have. A *quadrilateral* has four sides, and the interior angles always add up to 360°

SQUARE – four equal sides and four 90° angles
RECTRANGLE – four sides; opposite sides equal and parallel and four 90° angles
PARALLELOGRAM – four sides: opposite sides equal and parallel; no interior 90° angles; equal opposite angles
TRAPEZOID – four sides; two sides parallel and sides may be unequal

Polygon	**Perimeter Formula**	**Area Formula**
Square	P = 4s (sides)	$A = S^2$
Rectangle	P = 2l + 2w	A = lw
Parallelogram	P = 2l + 2w	A = bh
Trapezoid	P = a + b + c + d	$A = 1/2\ h\ (b_1 + b_2)$

Other polygons are the pentagon (five sides), hexagon (six sides), heptagon (seven sides), and octagon (eight sides).

EXERCISE 11

Circle the best possible answer.

1. What is the area of a triangle that measures 5 inches at the base and 7 inches in height?
   - a. 1.75 square inches
   - c. 17.5 square inches
   - b. 35 square inches
   - d. 170 square inches

2. What is the perimeter of an equilateral triangle if one side measures 19 inches?
   - a. 57 inches
   - c. 76 inches
   - b. 40.5 inches
   - d. 18 inches

3. What is the formula for finding the area of a rectangle?
   - a. $A = 1/2\ bh$
   - c. $A = lw$
   - b. $A = S^3$
   - d. none of the above

4. Polygons are
   - a. triangles
   - c. parallelograms
   - b. rectangles
   - d. all of the above

5. An eight-sided polygon is a
   - a. heptagon
   - c. hexagon
   - b. octagon
   - d. pentagon

6. A four-sided polygon which has opposite sides equal and parallel, has opposite angles equal, but does not have any interior angles of 90° is called a
   - a. square
   - c. trapezoid
   - b. triangle
   - d. parallelogram

7. A quadrilateral
   - a. has four sides
   - c. is a polygon
   - b. has interior angles totaling 360°
   - d. is all of the above

8. A square has a side of 13 inches. What is its area?
   - a. 78 sq in
   - c. 52 sq in
   - b. 169 sq in
   - d. cannot be determined

LESSON 12

Graphs

A graph is a visual representation of items compared to each other. It enables the reader to understand data more clearly. Some graphs actually show a picture or symbol of an item or a product, such as the following graph based on a government report for a four-year period.

Each symbol represents 2,000,000 boxes of grapefruit.

One can readily see that Arizona has one and one-half grapefruits. Therefore, Arizona produces 3,000,000 boxes of grapefruit. Texas has three-fourths of a grapefruit symbol. When we multiply ¾ by 2,000,000, we find that Texas produces 1,500,000 boxes.

Bar graphs and line graphs are used to represent comparisons. A bar graph utilizes a solid bar to indicate an amount. A line graph uses a line to connect points indicating amounts. The following illustrations show how the same information is treated in both graphs.

The circle graph represents the relationship of the parts to a whole thing. It is generally shown as a percentage. The *whole* represents 100%; all the parts add up to 100%. The graph shows the food elements in grapes.

EXERCISE 12

A. Use the grapefruit graph to answer the following questions.

1. How many boxes of grapefruit does Florida produce? ________________________________

2. How many times greater is the production of California than the production of Texas? ______

3. How many million boxes of grapefruit are produced by the four states? __________________

B. Use the bar and line graphs to answer the following questions.

1. What was the coldest month? ____

2. At what degree is the month of March represented on the bar graph? ____________________

3. At what degree is the month of March represented on the line graph? ____________________

C. Use the circle graph to answer the next group of questions.

1. What percent of the graph is not water? ____

2. Protein, fat, and ash make up what percent of the graph? _____________________________

3. What percent of the graph is made up of carbohydrates and water? ____________________

4. Which two components are closest in quantity? ____________________________________